Incorporating Two-Port Networks with S-Parameters into FDTD

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***Abstract*—A modeling approach for incorporating a two-port network with S-parameters in the finite-difference time-domain (FDTD) method is reported in this paper. The proposed method utilizes the time-domain Y-parameters to describe the network characteristics, and incorporates the Y-parameters into the FDTD algorithm. The generalized pencil-of-function (GPOF) technique is applied to improve the memory efficiency of this algorithm by generating a complex exponential series for the Y-parameters and using recursive convolution in the FDTD updating equations. A modeling example is given, which shows that this approach is effective and accurate. This modeling technique can be extended for incorporating any number of N-port networks in the FDTD modeling.**

***Index Terms*—FDTD method, generalized pencil-of-function (GPOF), S-parameters.**

I. Introduction

THE finite-difference time-domain (FDTD) method has been widely used to analyze many different types of electromagnetics problems. Work has been reported that extends the FDTD method to include passive and active elements in the modeling [1]–[3], where circuit elements such as resistors, capacitors, inductors, diodes, and transistors were treated as subgrid models on the FDTD mesh. Similar implementations of two-ports by time-domain convolution, and the insertion of lumped elements into distributed time-domain field models have been reported earlier for the TLM method [4]–[6]. An FDTD algorithm has also been proposed to simulate realistic devices with both active and nonlinear regions [7]. This algorithm used an extrapolated polynomial fit for the current–voltage relationship in the active and nonlinear region, and then expanded it in a Taylor series so that a finite difference representation of the equivalent circuit at the subcell level could be obtained. In [8], a lumped equivalent circuit of a microwave amplifier was used to characterize the current–voltage relationship of the device. Circuit theory was then applied to determine the node voltage by solving the state equation of the equivalent circuit at each time step. The voltage was then fed back into the FDTD simulation via an impressed current source $\vec{J}_{device}$ in the Ampere's current law,

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{H}}{\partial t} + \vec{J}_{device}. \tag{1}$$

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Another approach for including lumped circuit elements in FDTD modeling was reported in [9]. The methodology allows direct access to SPICE to model the lumped circuits, while the full 3-dimensional solution to Maxwell's equations provides the crosstalk and dispersive properties of the microstrip lines and striplines in the circuit. The FDTD and SPICE computer programs may be coupled using various interprocess communication techniques.

The work presented herein demonstrates an approach for incorporating a two-port network in the modeling if the S-parameters (or any parameter set that can be converted to Y- or Z-parameters) are provided. The method converts the S-parameters of the two-port network into Y-parameters, and uses an inverse fast Fourier transform (IFFT) to obtain the time-domain series of the Y-parameters. The generalized pencil-of-function (GPOF) technique is utilized to approximate the time-domain Y-parameters with a complex exponential series. The two-port network is incorporated in the FDTD modeling by specifying an impressed current source at each port. Using the experimental series results in a recursive convolution for the FDTD time-marching equations, which improves the memory efficiency. A modeling example is given to demonstrate the effectiveness and accuracy of this approach.

II. Incorporating a Two-Port Network With S-Parameters in the FDTD Algorithm

Simple networks may have an equivalent lumped element circuit model, and can be included in FDTD modeling using the algorithms reported in [2], [8]. However, a general algorithm that incorporates the network in the FDTD modeling can be very helpful. This is especially true for those cases where derivation of an equivalent circuit is difficult, or, the network is characterized by measurements.

The electric field time-stepping equation in the proposed algorithm is modified to allow for the addition of a two-port network. The Ampere's Law Maxwell equation becomes

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} + \vec{J}_{net} \tag{2}$$

where $\vec{J}_{net}$ is an impressed current density through which the network will be incorporated [3]. The network can also include nonlinearities and dispersion. Assuming the network is oriented along the z direction of the FDTD mesh, the current density is related to the node current as

$$J_{net} = \frac{I_{net}}{\Delta x \cdot \Delta y}. \tag{3}$$

At Port 1 of the network

$$J_{net1} = \frac{I_{net1}}{\Delta x \cdot \Delta y} = \frac{V_{net1} * Y_{11}(t) + V_{net2} * Y_{12}(t)}{\Delta x \cdot \Delta y} = \Delta z \cdot \frac{E_{net1} * Y_{11}(t) + E_{net2} * Y_{12}(t)}{\Delta x \cdot \Delta y} \quad (4)$$

where $*$ indicates convolution.

The time-stepping equation can then be readily developed from (2) and (4). In order to incorporate the network characteristics in the time domain, this approach converts the S-parameters into Y-parameters, then performs an IFFT to translate the Y-parameters into the time domain. The frequency-domain Y-parameters are further extended to $2 \times N$ by conjugating the first N steps before application of the IFFT in order to get a real time-domain sequence. The presence of the convolution in (4) requires storing the complete E-field and Y-parameter time histories, which increases memory usage. The GPOF technique is employed here [10], which uses a finite sum of complex exponentials to describe the time response sequences of the Y-parameters [$Y_{11}(n)$, $Y_{12}(n)$, $Y_{21}(n)$, and $Y_{22}(n)$], as

$$Y_{ij}(n) = \sum_{k=1}^{m} \alpha_{ij}(k)[\beta_{ij}(k)]^n \quad (5)$$

where $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ are the terms obtained by the GPOF method, and generally are complex numbers. For simplicity in the present development, the network is assumed located in free space, though this is not necessary. The time-marching equation for the z-component of the electric field at Port i ($i = 1$ or 2) is shown in (6) at the bottom of the page. Substituting (5) into (6) results in a recursive convolution for the time-marching equation, where

$$\sum_{l=0}^{n} Y_{ij}(l) \cdot E_{z,\,Port\,j}(n-l) = \sum_{k=1}^{m} \alpha_{ij}(k) \cdot E_{z,\,Port\,j}(n) + \sum_{k=1}^{m} \Phi_{ij}(n,\,k), \quad (7)$$

and

$$\Phi_{ij}(n,\,k) = \alpha_{ij}(k) \cdot \beta_{ij}(k) \cdot E_{z,\,Port\,j}(n-1) + \beta_{ij}(k) \cdot \Phi_{ij}(n-1,\,k). \quad (8)$$

Since the time response $Y_{ij}(n)$ is always real, the constants $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ always come in conjugate pairs if they are complex numbers, otherwise they are purely real. It can be shown then that the left-hand-side of (7) always results in a real number, which is necessary for the electric field updating equation in (6).

Application of the GPOF algorithm to generate the Y-parameters as an exponential series allows for recursive updating equations, where only the intermediate parameters $\Phi_{ij}(n,\,k)$ are stored for the current time step. These intermediate parameters are then overwritten at each following time step, without the need of storing the complete time history for the convolution. This treatment minimizes the computer memory usage. Fig. 1 shows the necessary data processing sequence for the network before incorporating it into the FDTD simulation. The constants $\alpha_{ij}(k)$ and $\beta_{ij}(k)$, which characterize the two-port network, are used as the input circuit description of the network for the FDTD simulation.

III. A Modeling Example

A simple microstrip circuit with a lumped-element network shown in Fig. 2 is used as an example to demonstrate the modeling algorithm. Two sections of microstrip line were connected by a low-pass filter that consists of several lumped elements. The value of each individual lumped element and the dimensions of the microstrip circuit are shown in the figure. The characteristic impedance of the microstrip line was $63\ \Omega$, as calculated using an empirical equation [11]. The $|S_{21}|$ was then modeled through two different approaches. The first approach used the traditional FDTD method with lumped element modeling algorithms as described in [2]. In the second modeling approach, the S-parameters for the lumped-element network were calculated, and the data processing procedures described in Fig. 1 were performed to obtain the constants $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ for the network. These constants, which described the characteristics of the network, were used as the input information of the two-port network instead of the lumped element values. Recursive convolution for the network was then used in the FDTD updating.

The FDTD cell size in the modeling was 1 mm $(x) \times$ 1 mm $(y) \times$ 0.5 mm (z) for both simulations. Eight perfectly matched layers (PML) were placed at the boundary of the computational domain to mimic an open-region problem, and seven white space layers were placed between the PML and the circuit. In this particular case, three cells were used to model the thickness of the dielectric, and the network port spanned only one cell. A connection was made to the trace and ground plane by adding two wires to span the other two cells of the board thickness to ensure current continuity. The connecting wires were modeled with a thin-wire algorithm [12]. The algorithm can be applied over multiple cells, but only a single cell is used here for simplicity. The modeled $|S_{21}|$ using the

$$E_{z,\,Porti}^{n+1} = E_{z,\,Porti}^{n} + \frac{\Delta t}{\varepsilon_0}\left[\nabla \times H^{n+(1/2)}\right]_z - \frac{\Delta t \cdot \Delta z \cdot \left[\sum_{l=0}^{n} Y_{i1}(l) \cdot E_{z,\,Port1}(n-l) + \sum_{l=0}^{n} Y_{i2}(l) \cdot E_{z,\,Port2}(n-l)\right]}{\varepsilon_0 \cdot \Delta x \cdot \Delta y} \quad (6)$$

$$\begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \xrightarrow{\text{S to Y}} \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) \\ Y_{21}(\omega) & Y_{22}(\omega) \end{bmatrix} \xrightarrow{\text{IFFT}} \begin{bmatrix} Y_{11}(t) & Y_{12}(t) \\ Y_{21}(t) & Y_{22}(t) \end{bmatrix} \xrightarrow{\text{GPOF}} \begin{bmatrix} \{\alpha_{11}^k, \beta_{11}^k\} & \{\alpha_{12}^k, \beta_{12}^k\} \\ \{\alpha_{21}^k, \beta_{21}^k\} & \{\alpha_{22}^k, \beta_{22}^k\} \end{bmatrix}$$

Fig. 1. Data processing sequence to incorporate S-parameters into the FDTD modeling.

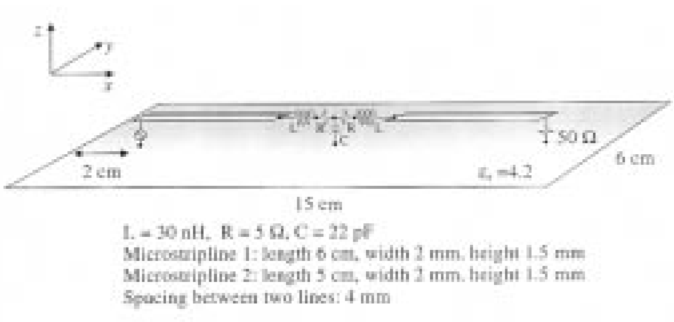

Fig. 2. Microstrip circuit example to demonstrate the algorithm for incorporating a two-port network in the FDTD modeling.

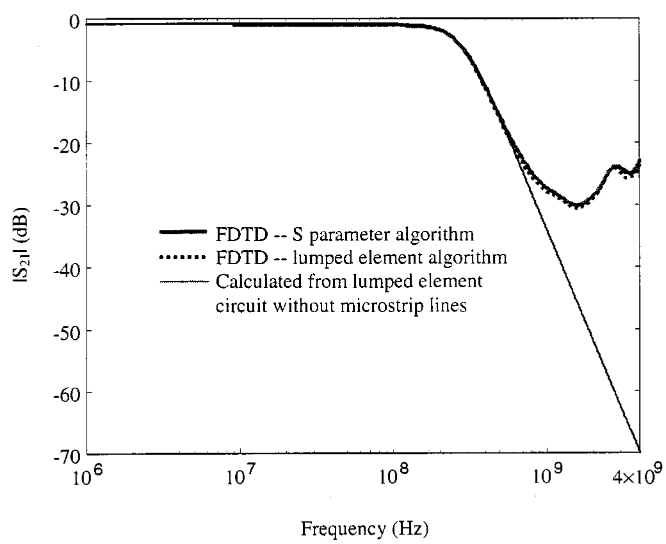

Fig. 3. FDTD modeled $|S_{21}|$ for the circuit shown in Fig. 2.

lumped-element and S-parameters approaches is shown in Fig. 3. The $|S_{21}|$ directly calculated from the lumped-element circuit without the microstrip lines is also shown in the same figure. The results indicate that the distributed behavior of the circuit is manifested at frequencies greater than 600 MHz. Good agreement was achieved between the results using the two different modeling approaches. This indicates that the proposed algorithm is feasible and accurate for incorporating a two-port network in the FDTD modeling using S-parameters. The S-parameters can be obtained from calculation, measurements, or data sheets, etc.

The required CPU time for both the lumped-element and S-parameter approaches is comparable unless there are a large number of networks in the computational domain. Otherwise, the computational time is dominated by the general scattering and PML time-marching equations. However, the new modeling approach has the advantage of ease of implementation for a more complex network, in particular, for those cases that an equivalent circuit model is not available. The GPOF technique and recursive convolution can also significantly reduce the memory requirement for a large number of networks. Another advantage of this method is that it can be easily extended to include multiport networks with S-parameters, since for an N-port network, the impressed current at Port i for (2) can be written as

$$I_{neti} = \sum_{j=1}^{N} Y_{ij} \cdot V_j \tag{9}$$

and derivation of the time marching equations similar to (6) is straightforward.

IV. Conclusion

An approach for incorporating a two-port network through S-parameters in FDTD modeling is reported in this paper. The method uses the time-domain Y-parameters to describe the network characteristic, and incorporates them into the FDTD algorithm. The GPOF technique is applied in order to do the convolution recursively, which improves the memory efficiency. A two-port microstrip circuit example demonstrates this approach. Further, the reported modeling approach can be readily extended to incorporate any number of N-port networks in the modeling.

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